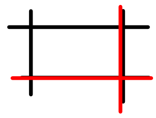
§ Uhlenbeck spaces

- factorization

  fix a projection $\mathbb{A}^2 \to \mathbb{A}^1$

$$\pi_a : \mathcal{U}_G^d \to S^d \mathbb{A}^1$$

$$\mathcal{U}_G^d = \coprod_{d',\lambda} \mathrm{Bun}_G^{d'} \times \underline{S_\lambda \mathbb{A}^2} \xrightarrow{a}$$

- $\mathrm{Bun}_G^d = \{(E, \varphi) \mid \mathbb{P}^1 \times \mathbb{P}^1 \text{ trivialised at }$  $\}$

$$\mathbb{P}^1 \times \mathbb{P}^1 \xrightarrow{a} \mathbb{P}^1$$

$a^{-1}(x) \cong \mathbb{P}^1$ $\qquad E|_{a^{-1}(x)}$: trivial when $x = \infty$
└ open condition

$\Rightarrow$ $E|_{a^{-1}(x)}$: nontrivial at $x = x_1, \cdots, x_\ell$

If we count with multiplicities,

$\Rightarrow$ # of pts $= d$

- fiber $\qquad \mathrm{Spec}(a_1 B_1 + a_2 B_2)$

$$a = [a_1 : a_2]$$

Alternative description : $\mathrm{Bun}_G^d \cong \mathrm{Map}_d(\mathbb{P}^1, \mathrm{Gr}_G)$
└ affine Grassmann
space of based maps

Factorization property :

$\pi_a(E, \varphi)$: disjoint $\Longrightarrow$ $\mathcal{U}_G^d \sim \mathcal{U}_G^{d_1} \times \cdots \times \mathcal{U}_G^{d_\ell}$ locally